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Super Yang-Mills with flavors from large-N_f matrix models

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Abstract: We consider the exact effective superpotential of $\mathcal{N} = 1$ U(N_c) super Yang-Mills theory with N_f massive flavors an additional adjoint Higgs field. We use the proposal of Dijkgraaf and Vafa to calculate the superpotential in terms of a matrix model with a large number of flavors. We do this by gauging the flavor symmetry and forcing this sector in a classical vacuum. This gives rise to a 2-matrix model of ADE type A_2, and large flavors. This approach allows us to add an arbitrary polynomial tree level superpotential for the Higgs field, and use strict large-N methods in the matrix model.

Keywords: Nonperturbative Effects, 1/N Expansion, M(atrix) Theories.

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1. Introduction

Recently, Dijkgraaf and Vafa (DV) proposed an exact calculation of superpotentials in $\mathcal{N} = 1$ super Yang-Mills theories using matrix models [1, 2, 3]. Later the derivations of the relation to the matrix models was shown directly from the field theory in [4, 5]. The DV proposal has initiated much research, extending the proposal to include flavors, other gauge groups, gravitational corrections, and relation to Seiberg Witten curves and field theory calculations of the effective superpotential [6]–[43].

The basic proposal of DV considers an $\mathrm{U}(N_c)$ gauge theory with an adjoint Higgs field Φ, which has a tree level superpotential $\mathrm{Tr}\, W(\Phi)$. The solution is given in terms of a matrix model with of a $M \times M$ matrix Φ with action $\frac{1}{g_s} \mathrm{Tr}\, W(\Phi)$. The effective superpotential for the glueball field $S = \langle \lambda\lambda \rangle$ can be calculated as the derivative of the planar contribution to the free energy in the matrix model

$$W_{\mathrm{eff}}(S) = N_c \frac{\partial \mathcal{F}_0}{\partial S}\,, \tag{1.1}$$

identifying S with the 't Hooft coupling $g_s M$ of the matrix model. As only the planar diagrams contribute, this can be calculated by large-N techniques.

In the approach to the addition of flavors, there are two proposals. First in [6] it is conjectures that one has to add the planar free energy with 1 quark-loop to the above effective action. This proposal has been used in most of the other approaches to the use of matrix models for super Yang-Mills with flavors, and gives agreement with previously known results. The proposal of [7] amounts to taking also the number of flavors M_f in the matrix model to infinity, but holding the ratio M_f/M fixed. One can easily convince oneselve that this gives contributions from all quark loops, so the proposals seem to be incompatible. One can argue that there should only contributions from planar diagrams with at most one quark loop to the effective superpotentials. This is a generalization of the arguments used in [1, 5] to show that the effective superpotential has only contributions from the planar diagrams and is linear in N_c. What makes the proposal of [6] harder to work with is the fact that one needs $1/N$ corrections. These could in principle be calculated using techniques developed in [44, 45]. In most other papers on the subject only quadratic tree level superpotentials were considered, for which the planar diagrams can be summed explicitly.

In this paper we will show that one can add flavors to DV staying purely in a large-N limit. As in [7] we consider a matrix model with a large number of flavors M_f, but we will not constrain their ratio. This would add an extra large-N parameter $S_f = g_s M_f$, which we will take to zero at the end of the calculation. The proposed formula for the effective superpotential is then

$$W_{\text{eff}}(S) = N_c \frac{\partial \mathcal{F}_0}{\partial S} + N_f \frac{\partial \mathcal{F}_0}{\partial S_f} \,. \tag{1.2}$$

A simple counting of the dependence of the contributions on S and S_f shows that there are only contributions of the planar diagrams. Taking $S_f = 0$ will then give only the contributions with at most one quark loop. Hence our proposal reproduces [6], but in a large-N context.

Another way to study this system using only the original DV proposal can be found by gauging the flavor symmetry. For this we consider a $\mathrm{SU}(N_c) \times \mathrm{SU}(N_f)$ super Yang-Mills with bifundamentals and adjoint Higgs fields for each of the groups. The effective superpotential for this model can be calculated by an ADE type 2-matrix model as in [2, 46]. The form of the effective superpotential has roughly the form (1.2) above. The $\mathrm{SU}(N_c)$ theory can be extracted by putting the $\mathrm{SU}(N_f)$ in a classical vacuum. Note that in this system $S_f = g_s M_f$ becomes the gluino condensate in the $\mathrm{SU}(N_f)$ factor. The $\mathrm{SU}(N_f)$ Higgs field then will play the role the mass matrix for the quarks. This can be accomplished by adding and putting the $\mathrm{SU}(N_f)$ glueball field S_f to zero. As $S_f = g_s M_f$ in the matrix model this corresponds nicely to what we had above. This system can be understood in terms of string theory by wrapping D5-branes on two different 2-spheres intersecting at a point, where one of the two spheres is taken very large, freezing out the dynamics of the D-branes wrapping it. From this point of view this was also studied recently in [14].

This paper is organized as follows. In section 2 we discuss the matrix model approach and show in more detail the genus expansion for the different proposals. In section 3 we study the 2-matrix model of type A_2 and relate it to the flavor model. In section 4 we work out explicitly the model with quadratic superpotential and compare with another approach using meson potentials. In section 5 we conclude with same discussion.

2. Flavors and matrix models

2.1 Matrix models with flavors

We consider an $\mathrm{SU}(N_c)$ gauge theory with N_f pairs of quark fields Q_i and $\tilde{Q}_i$, $i = 1, \cdots, N_f$, in the fundamental and the anti-fundamental of $\mathrm{SU}(N_c)$ respectively. Note that the gauge superfield W_α contains a complex scalar Φ in the adjoint representation of the color group.

The lagrangian density of this theory is given by

$$L = \int d^4\theta \, \mathrm{Tr}(\bar{Q}_i \mathrm{e}^V Q_i + \tilde{Q}_i \mathrm{e}^V \bar{\tilde{Q}}_i) + \int d^2\theta \, \mathrm{Tr}(W(\Phi, Q, \tilde{Q}) + \tau W^\alpha W_\alpha) \,, \tag{2.1}$$

with superpotential given by

$$W(\Phi, Q, \tilde{Q}) = W(\Phi) + \Phi Q \tilde{Q} - Q m \tilde{Q} \,, \tag{2.2}$$

where the first term is a polynomial tree level superpotential for the adjoint Higgs field Φ and m is the mass matrix for the flavors. Classically the eigenvalues of the Higgs field and the quarks will be at critical points of this tree level superpotential. This breaks the gauge group to $\times_I \mathrm{SU}(N_I)$ where N_I is the number of eigenvalues of Φ at the critical point I-th critical point. Quantum mechanically there will be a gluino condensate S_I for each of the factors. The goal is to find the effective superpotential for these condensates.

The effective superpotential can be calculated according to the proposal of Dijkgraaf and Vafa, using matrix models. Here we take the superpotential of the gauge theory reduced to a point in superspace, replacing the number of colors N_c by the matrix model size M of the number of flavors N_f by M_f. So the fields are an $M \times M$ matrix Φ, an $M \times M_f$ matrix Q and a $M_f \times M$ matrix $\tilde{Q}$. The partition sum of the matrix model is given by

$$Z = \mathrm{e}^{\mathcal{F}} = \frac{1}{Vol(G)} \int \mathcal{D}\Phi \mathcal{D}Q \mathcal{D}\tilde{Q} \, \mathrm{e}^{-\frac{1}{g_s} \mathrm{Tr} \, W(\Phi, Q, \tilde{Q})} \,, \tag{2.3}$$

where W is the same function as above, but now of the matrix variables, and G is the unbroken gauge group.

We denote by g the genus of the diagram, and by h the number of quark loops. By this we mean the number of boundary components coupling to the flavor matrices. Note that this is not the same as the number of holes (i.e. the number of holes coupling to the gauge multiplet). The dependence of the free energy of the parameters g_s, M, and M_f can easily be extracted from the topology of the diagrams and can be written as an expansion in the genus and the number of quark loops as

$$\mathcal{F} = \sum_{g,h} g_s^{2g+h-2} M_f^h \mathcal{F}_{g,h}(g_s M) = \sum_{g,h} g_s^{2g-2} (g_s M_f)^h \mathcal{F}_{g,h}(g_s M) \,. \tag{2.4}$$

Let us therefore introduce parameters $S \equiv g_s M$ and $S_f \equiv g_s M_f$.

We propose, analogous to DV, that the effective superpotential can be found from an expression of the form

$$W_{\mathrm{eff}}(S) = N_c \frac{\partial \mathcal{F}_0}{\partial S}(S, S_f = 0) + N_f \frac{\partial \mathcal{F}_0}{\partial S_f}(S, S_f = 0) \,, \tag{2.5}$$

where $\mathcal{F}_0$ is the planar contribution to the free energy for genus 0. This is a generalization of the pure gauge theory studied by Dijkgraaf and Vafa.

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The planar contributions can be found from a large-M expansion of the matrix model. All planar diagrams are summed by taking both the rank of the gauge group and the number of flavors to infinity,

$$M,\, M_f \to \infty\,, \qquad g_s \to 0\,, \qquad S = g_s M\,, \quad S_f = g_s M_f \quad \text{finite}\,. \tag{2.6}$$

From the expansion above we see that this picks out the genus zero contribution.

Let us relate the proposal above to the calculation of the matrix model in a large-M limit taking $M_f = N_f$ fixed, as in [6] and many papers following it. From the above expansion we find in the large-M limit

$$N_c \frac{\partial \mathcal{F}_0}{\partial S} + N_f \frac{\partial \mathcal{F}_0}{\partial S_f} = \sum_{h \geq 0} \left(N_c S_f^h \frac{\partial \mathcal{F}_{0,h}(S)}{\partial S} + h N_f S_f^{h-1} \mathcal{F}_{0,h}(S) \right). \tag{2.7}$$

Taking $S_f = 0$ there are only two surviving terms: in the first sum we only get the $\mathcal{F}_{0,0}$ contribution, while in the second sum we only retain $\mathcal{F}_{0,1}$. Therefore the effective superpotential is given by

$$W_{\text{eff}}(S) = N_c \frac{\partial \mathcal{F}_{0,0}}{\partial S}(S) + N_f \mathcal{F}_{0,1}(S) = N_c \frac{\partial \mathcal{F}_{\chi=2}}{\partial S}(S) + \mathcal{F}_{\chi=1}(S)\,. \tag{2.8}$$

We find the same expression as [6]. The advantage of our proposal is that in keeping $M_f = N_f$ finite, the contribution at $\chi = 1$ is subleading in g_s or equivalently $1/M$. Therefore one cannot completely rely on large-M expansion in the matrix model, which makes it harder to do actual computations, especially if one wants to consider general tree level superpotentials for the adjoint Higgs field.

We should note that, as opposed to the proposal of [7], we do not relate M_f to N_f. Particularly, the ration M_f/M is not fixed in the large-M limit. This is necessary, as we want to take $S_f = 0$ at the end. Indeed, the matrix model quantities M and M_f are completely unrelated to the gauge theory quantities N and N_f.

2.2 Loop equations

Let us first consider the matrix model with a large number of flavors, as described above. To calculate the matrix model integral, we integrate out the fundamentals. We are then left with an integral over Φ,

$$Z = \int \mathcal{D}\Phi\, \mathrm{e}^{-\frac{1}{g_s} \operatorname{Tr} V(\Phi)}\,, \tag{2.9}$$

where

$$V(\Phi) = W(\Phi) + S_f \log(\Phi - m)\,. \tag{2.10}$$

We take the large-M, M_f limit with $g_s \to 0$ such that the 't Hooft coupling $S = g_s M$ and $S_f = g_s M_f$ is fixed.

As is usual we introduce the resolvent defined as

$$\omega(x) = \frac{1}{M} \operatorname{Tr} \frac{1}{x - \Phi}\,. \tag{2.11}$$

We can then derive the loop equation

$$\oint_C \frac{dz}{2\pi i} \frac{V'(z)}{x-z} \langle \omega(z) \rangle = S \langle \omega(x)^2 \rangle \,, \tag{2.12}$$

as a Schwinger-Dyson equation for the matrix model. Here we have to be careful that C is a cycle enclosing all the cuts but not the point x or m. The latter is because $V'(z)$ has a pole at $z = m$. This loop equation can also be written in the form

$$S \langle \omega(x)^2 \rangle - V'(x) \langle \omega(x) \rangle + \frac{1}{4S} f(x) = 0 \,, \tag{2.13}$$

where

$$f(x) = \frac{4S}{M} \left\langle \mathrm{Tr} \, \frac{V'(x) - V'(\Phi)}{x - \Phi} \right\rangle . \tag{2.14}$$

Note that if V were polynomial, this would be a polynomial in x. Also, in the large-M limit we have $\langle \omega(x)^2 \rangle = \langle \omega(x) \rangle^2 + \mathcal{O}(\frac{1}{M^2})$. Therefore we can replace $\omega(x)$ by its expectation value. Introducing $y(x) = -2S\omega(x) + V'(x)$ we find the spectral curve

$$y^2 = V'(x)^2 - f(x) \,. \tag{2.15}$$

Unlike the usual matrix model however, the functions $V'(x)$ and $f(x)$ have a pole at $x = m$. The pole term in $f(x)$ is given by

$$\frac{4S}{M} \left\langle \mathrm{Tr} \left(\frac{1}{x - \Phi} \left(\frac{S_f}{x-m} - \frac{S_f}{\Phi - m} \right) \right) \right\rangle = \frac{4SS_f}{x-m} \omega(m) \,. \tag{2.16}$$

As a result a multi-cut solution has the form

$$y(x) = \left(M(x) + \frac{\gamma}{x-m} \right) \sqrt{\prod_i (x - x_i)} \,, \tag{2.17}$$

where the cuts lie along the intervals $[x_{2i-1}, x_{2i}]$, $M(x)$ is a polynomial with degree determined by the number of cuts and the degree of W, and γ is determined by the pole at $x = m$ to be

$$\gamma = \frac{S_f}{\sqrt{\prod_i (m - x_i)}} \,. \tag{2.18}$$

Note that this coefficient is also such that the resolvent

$$2S\omega(x) = W'(x) + \frac{S_f}{x-m} - y(x) \,, \tag{2.19}$$

has no pole at $x = m$.

Rather than pursuing this direction, we want to proceed to a different method, relating the model to a special limit of a 2-matrix model. As it turns out several aspects of the solution can be seen much more clearly in this approach.

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3. Flavors from the 2-matrix model

In this section we discuss the use of the 2-matrix model to calculate the effective superpotential. We will interpret the two gauge group as the color and flavor symmetry group, which now are both gauged. To find the $\mathrm{SU}(N_c)$ theory with fundamental flavors we take a limit forcing the flavor sector into a classical vacuum.

3.1 The A_2 Model

We propose a description of the flavor model in terms of certain multi-matrix models. In particular, the A_2 model. The idea is as follows. The A_2 model is based on a quiver with 2 nodes, both corresponding to a gauge group $\mathrm{U}(N_i)$. We have two bifundamentals, which we call $Q = Q_{12}$ and $\tilde{Q} = Q_{21}$. The tree level superpotential is given by

$$\mathrm{Tr}_1\, W_1(\Phi_1) + \mathrm{Tr}_2\, W_2(\Phi_2) + \mathrm{Tr}_1\, \tilde{Q}\Phi_1 Q - \mathrm{Tr}_2\, Q\Phi_2\tilde{Q}\,. \tag{3.1}$$

Let us discuss the solution of this model. Part of the following discussion, and some more details, can also be found in [2]. We can write the matrix integrals in terms of eigenvalues $\lambda_{i,I}$ of the matrices Φ_i. We introduce the resolvents for the two adjoints,

$$\omega_i(x) = \frac{1}{M_i}\,\mathrm{Tr}_i\,\frac{1}{x-\Phi_i} = \frac{1}{M_i}\sum_I \frac{1}{x-\lambda_{i,I}}\,. \tag{3.2}$$

After integrating out the bifundamentals $Q, \tilde{Q}$, we obtain [2]

$$\int \prod_{i,I} d\lambda_{i,I} \prod_{(i,I)\neq(j,J)} (\lambda_{i,I}-\lambda_{j,J})^{C_{ij}}\, \mathrm{e}^{-\frac{1}{g_s}\sum_{i,I} W_i'(\lambda_{i,I})}\,. \tag{3.3}$$

with $C_{ij} = 2\delta_{i,j} - \delta_{i,j-1} - \delta_{i,j+1}$ the Cartan matrix of the A_2 model. The forces, or equivalently equations of motion, can be written

$$y_i(x) = -g_s\partial_{\lambda_{i,I}} S(x) = W_i'(x) - C_{ij}S_j\omega_i(x)\,, \tag{3.4}$$

where $S_i = g_s M_i$ are the 't Hooft couplings. It is convenient to introduce two scalar fields $\varphi_i(x)$ on the x-plane related to the forces as $\partial\varphi_i(x) = y_i(x)dx$. The matrix integral can in fact be related to the rational CFT of type A_2 in terms of these scalar fields. This CFT is known to have a structure of a W_3 algebra formed by 2 currents $\mathcal{W}^{(s)}(x)$ of spins $s = 2, 3$. These currents also exist in the matrix model [46].

To write expressions for the currents, it is convenient to use a slightly different basis for the forces [2],

$$\begin{aligned} a_1(x) &= t_1(x) - S_1\omega_1(x)\,, \\ a_2(x) &= t_2(x) + S_1\omega_1(x) - S_2\omega_2(x)\,, \\ a_3(x) &= t_3(x) + S_2\omega_2(x)\,, \end{aligned} \tag{3.5}$$

where the t_i are the classical contributions

$$
\begin{aligned}
t_1(x) &= \frac{2}{3}W_1'(x) + \frac{1}{3}W_2'(x)\,, \\
t_2(x) &= -\frac{1}{3}W_1'(x) + \frac{1}{3}W_2'(x)\,, \\
t_3(x) &= -\frac{1}{3}W_1'(x) - \frac{2}{3}W_2'(x)\,.
\end{aligned} \tag{3.6}
$$

These are not independent, but satisfy $\sum_i a_i = 0 = \sum_i t_i$. The two spin s currents can then be given by [46]

$$
\mathcal{W}^{(s)}(x) = \frac{(-1)^s}{s}\sum_{i=1}^{3} a_i(x)^s\,, \qquad s = 2, 3\,. \tag{3.7}
$$

They have a classical contribution $\mathcal{W}^{(s)}_{cl}(x)$ which can be found by replacing the a_i with the t_i. These currents satisfy loop equations of the form

$$
\oint_C \frac{dz}{2\pi i}\,\frac{1}{x-z}\langle \mathcal{W}^{(s)}(z)\rangle = 0\,. \tag{3.8}
$$

where C is a cycle enclosing all the eigenvalues but not the point x. The loop equations imply that

$$
\langle \mathcal{W}^{(s)}(x)\rangle - \mathcal{W}^{(s)}_{cl}(x) = w^{(s)}(x)\,, \tag{3.9}
$$

where $w^{(s)}(x)$ are regular functions of x. If the $W_i(x)$ are polynomials of degree $n+1$, one can show that $w^{(s)}$ is a polynomial of degree $ns - n - 1$. In the large-M limit the expectation values factorize, so we do not have to distinguish between the resolvent as an operator and its expectation value in the expression for the current.

The quantum spectral curve of this model can be written

$$
\prod_{i=1}^{3}(y - a_i(x)) = 0\,. \tag{3.10}
$$

Using the currents we can write this as a deformation of the classical curve — the above curve with the a_i replaced by the t_i —

$$
\prod_{i=1}^{3}(y - t_i(x)) + y w^{(2)}(x) + w^{(3)}(x) = 0\,. \tag{3.11}
$$

This quantum curve can now be conveniently used to study the large-M behavior of the model.

3.2 Free energy at large M and the effective superpotential

We will now study the large-M limit by looking at the shape of the quantum curve. Let us first study the classical equations of motion of the system. In the following we will assume that there are no simultaneous solutions to $W_1'(\lambda) = 0$ and $W_2'(\lambda) = 0$. This is not a strong condition and will be sufficient for the rest of this paper.

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We can use the $\mathrm{U}(N_1) \times \mathrm{U}(N_2)$ symmetry to diagonalize the matrices Φ_i; their eigenvalues will be denoted $\lambda_{i,I}$. For the corresponding blocks Q_{IJ} and $\tilde{Q}_{JI}$ in Q and $\tilde{Q}$ the equations of motion give

$$(\lambda_{1,I} - \lambda_{2,J})Q_{IJ} = 0\,, \qquad (\lambda_{1,I} - \lambda_{2,J})\tilde{Q}_{JI} = 0\,. \tag{3.12}$$

This implies that the blocks Q_{IJ} and $\tilde{Q}_{JI}$ are nonzero only if $\lambda_{1,I} = \lambda_{2,J}$. We can therefore safely reduce the discussion to individual eigenvalues and henceforth suppress the extra index I.

The remaining Φ_i equations of motion are given by

$$W_1'(\lambda_1) = -Q\tilde{Q}\,, \qquad W_2'(\lambda_2) = \tilde{Q}Q\,, \tag{3.13}$$

where the left-hand sides have to be interpreted as diagonal matrices. There are three types of solutions to these equations.

When a block in $Q\tilde{Q}$ is zero, we find a solution for a critical point of the first node, $W_1'(\lambda_1) = 0$. We will call these critical points $e_{1,I}$. Similarly, we have solutions $e_{2,I}$ corresponding to the critical points of the second node, $W_2'(\lambda_2) = 0$. It remains to study the critical points when the eigenvalues are equal, $\lambda_1 = \lambda_2 \equiv \lambda_{12}$. It follows from the equations of motion (3.13) and some elementary algebra that $Q\tilde{Q} = \tilde{Q}Q$.[1] Therefore, these eigenvalues satisfy

$$W_1'(\lambda_{12}) + W_2'(\lambda_{12}) = 0\,. \tag{3.14}$$

The solutions to this equation will be called $e_{12,I}$.

The three different types of classical solutions can be related to the intersections of the three sheets comprising the classical curve. The intersection of the i-th and j-th sheet is given by the solutions to $t_i(x) - t_j(x) = 0$.

Quantum mechanically, the eigenvalues for Φ_i will spread around these classical solutions. In the large-M limit they will form cuts in the complex x-plane. The contours around the cuts will be denoted $A_{i,I}$ and $A_{12,I}$. The resulting quantum curve is sketched in figure 1; the 3 sheets correspond to the solutions $y = a_i(x)$. We also indicated the positions of the cuts around the different classical solutions.

The number of eigenvalues, or rather the 't Hooft couplings $S_{i,I} = g_s M_{i,I}$, can be measured by the contour integrals around the cuts,

$$S_{i,I} = -\frac{1}{4\pi i}\oint_{A_{i,I}} y(x)dx\,, \quad i = 1,2\,; \qquad S_{12,I} = -\frac{1}{4\pi i}\oint_{A_{12,I}} y(x)dx\,. \tag{3.15}$$

Integrating at infinity on the first and third sheet, we find that the total 't Hooft couplings $S_i = g_s M_i$ are given by the sums

$$S_1 = \sum_I S_{1,I} + \sum_I S_{12,I}\,, \qquad S_2 = \sum_I S_{2,I} + \sum_I S_{12,I}\,. \tag{3.16}$$

[1] Assume, w.l.o.g., that $W_1'(\lambda_{12}) \neq 0$. Then we can write, after a basis transformation, $Q = \begin{pmatrix} Q_1 & 0 \end{pmatrix}$ and $\tilde{Q} = \begin{pmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{pmatrix}$, with Q_1 and $\tilde{Q}_1$ invertible matrices. We find $\tilde{Q}Q \neq 0$ and, using (3.13), it must be that $Q = Q_1$. Therefore $\tilde{Q}Q = Q^{-1}(Q\tilde{Q})Q = Q\tilde{Q}$, as $Q\tilde{Q}$ is central.

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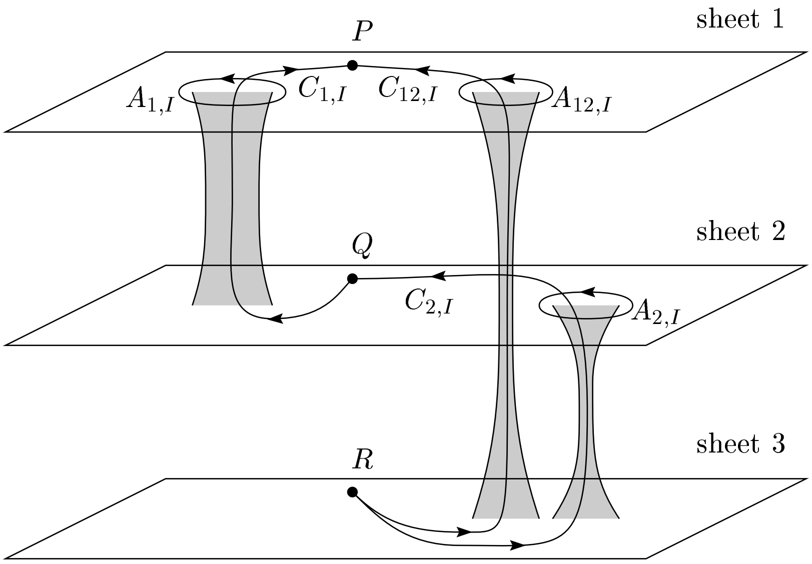

Figure 1: The three sheets of the A_2 curve, and the three cuts connecting them. The points P, Q, R are the points at regularized infinity.

For our purpose we need to solve the 2-matrix model in the large-M limit. At large M we can replace the sums over eigenvalues by the integral over eigenvalue distributions $\rho_1(\lambda_1)$ and $\rho_2(\lambda_2)$ respectively. They can be found from the discontinuity in the corresponding resolvents. The large-M limit of the free energy can be found from the saddle point approximation,

$$\mathcal{F}_0(S_1, S_2) = -S_i \int d\lambda_i\, \rho_i(\lambda_i) W_i(\lambda_i) + \frac{1}{2} C_{ij} S_i S_j \int d\lambda_i d\lambda_j\, \rho_i(\lambda_i)\rho_j(\lambda_j) \log(\lambda_i - \lambda_j)\,. \quad (3.17)$$

As in [1], the free energy can be found from the curve by considering the change of the density when we remove an eigenvalue from the cuts. When we remove an eigenvalue from the cut at $x = e_{i,I}$, the density changes by

$$\delta_{i,I}(S_i\rho_i)(\lambda_i) = \delta S_{i,I}\delta(\lambda_i - e_{i,I})\,, \qquad i = 1, 2\,. \quad (3.18)$$

Using this we find, for $i = 1, 2$, a variation

$$\frac{\partial \mathcal{F}_0}{\partial S_{i,I}} = -W_i(e_{i,I}) + C_{ij} S_j \int d\lambda_j\, \rho_j(\lambda_j) \log(\lambda_j - e_{i,I}) = \int_{e_{i,I}}^{\infty} (a_i(x) - a_{i+1}(x)) dx\,. \quad (3.19)$$

As $y(x) = a_i(x)$ on the i-th sheet, we can write this as a contour integral over the noncompact curve $C_{i,I}$ going through the appropriate cut, as indicated in figure 1,

$$\frac{\partial \mathcal{F}_0}{\partial S_{i,I}} = \int_{C_{i,I}} y(x) dx\,. \quad (3.20)$$

As the curves are noncompact we should regularize these integrals by ending the curves at finite points P, Q and R on the three sheets respectively. A similar discussion holds when we remove an eigenvalue at $e_{12,I}$. We now find the integral over $a_1(x) - a_3(x)$, which can be written as an integral over the noncompact curve $C_{12,I}$ connecting the first and third sheet,

$$\frac{\partial \mathcal{F}_0}{\partial S_{12,I}} = \int_{C_{12,I}} y(x)dx\,. \tag{3.21}$$

Let us now relate these to the effective superpotential of the quiver gauge theory. The original $\mathrm{SU}(N_1) \times \mathrm{SU}(N_2)$ gauge group will be broken by the distribution of the eigenvalues of the adjoint Higgs fields around the different classical vacua. The ranks therefore decompose analogously to

$$N_1 = \sum_I N_{1,I} + \sum_I N_{12,I}\,, \qquad N_2 = \sum_I N_{2,I} + \sum_I N_{12,I}\,, \tag{3.22}$$

analogous to (3.16). Here the $\mathrm{SU}(N_{12,I})$ factors are embedded diagonally in both original group factors. The effective potential for the A_2 quiver theory should now be given in terms of the matrix theory quantities by

$$W_{\text{eff}} = N_{1,I}\frac{\partial \mathcal{F}_0}{\partial S_{1,I}} + N_{2,I}\frac{\partial \mathcal{F}_0}{\partial S_{2,I}} + N_{12,I}\frac{\partial \mathcal{F}_0}{\partial S_{12,I}}\,. \tag{3.23}$$

3.3 Decoupling the flavor group

To relate the A_2 model to the flavor model, we identify the group $\mathrm{SU}(N_2)$ with the flavor group. To decouple this group we need to take this sector in a classical vacuum. Doing so, the adjoint Φ_2 of this node will play the role of the mass matrix m. To get the right masses we take the tree level superpotential of the flavor node such that

$$W_2'(\Phi_2) = \kappa \prod_{a=1}^{N_f}(\Phi_2 - m_a)\,. \tag{3.24}$$

We will then take $\kappa \to \infty$. This ensures that the flavor node will not add any quantum corrections, so that indeed Φ_2 will be constrained to the critical points m_a.

So we have to consider the limit $\kappa \to \infty$ of the cubic Riemann surface. In this limit the Riemann surface degenerates, and the third sheet will decouple. Note that for large κ, all the cuts in sheet 3 corresponding to $y = a_3(x)$ are around the points $x = m_a$, as W_2' dominates. For large κ around $x = m_a$, the resolvent $S_2\omega_2(x)$ will have the form

$$S_2\omega_2(x) \sim \frac{1}{2}\kappa_a(x - m_a) - \frac{1}{2}\sqrt{\kappa_a^2(x-m_a)^2 - 4\kappa_a S_{2,a}}\,, \tag{3.25}$$

where κ_a is proportional to κ. κ_a and $S_{2,a}$ in the above equation will depend on x, but have finite values at $x = m_a$. Taking large κ we have

$$S_2\omega_2(x) \sim \frac{1}{2}\kappa_a(x - m_a) - \frac{1}{2}\sqrt{\kappa_a^2(x-m_a)^2 - 4\kappa_a S_{2,a}} = \frac{S_{2,a}}{x - m_a} + \mathcal{O}\left(\frac{1}{\kappa}\right). \tag{3.26}$$

Therefore in the limit $\kappa \to \infty$ we have

$$S_2\omega_2(x) = \sum_{a=1}^{N_f}\frac{S_{2,a}}{x - m_a}\,. \tag{3.27}$$

The above arguments would also allow extra regular terms, but the asymptotics $\omega_2(x) \sim \frac{1}{x}$ for large x does not allow them. Therefore the above form is exact in the limit $\kappa \to \infty$. Of course this was expected; the eigenvalues of Φ_2 are all in their classical vacua at $\lambda_2 = m_a$.

We see that in the limit the branch cut dissapears, therefore the equation for the Riemann surface factorizes in this limit. We should then be interested in the part coming from the other two sheets. We have to change the coordinate y to absorb the large κ part. The proper shifted coordinate on the Riemann surface can be given by

$$\tilde{y} = 2y + a_3(x)\,. \tag{3.28}$$

We shift the $a_{1,2}$ by the same amount,

$$\tilde{a}_1 = 2a_1 + a_3\,, \qquad \tilde{a}_2 = 2a_2 + a_3\,. \tag{3.29}$$

Note that we now have $\tilde{a}_1 + \tilde{a}_2 = 0$. The equation $(y - a_1)(y - a_2) = 0$ for the decoupled first two sheets becomes

$$(\tilde{y} - \tilde{a}_1)(\tilde{y} - \tilde{a}_2) = \tilde{y}^2 - (W_1'(x) + S_2\omega_2(x))^2 + f(x)\,. \tag{3.30}$$

The quantum correction

$$f(x) = -4S_1^2\omega_1^2 + 4S_1\omega_1 W_1' + 4S_1S_2\omega_1\omega_2\,, \tag{3.31}$$

is a rational function with single poles at $x = m_a$. This can be seen by writing this in terms of the quantum contribution to the spin 2 current,

$$S_1^2\omega_1(x)^2 - S_1S_2\omega_1(x)\omega_2(x) - S_1\omega_1(x)W_1'(x) = w^{(2)}(x) - S_2^2\omega_2(x)^2 + S_2\omega_2(x)W_2'(x)\,, \tag{3.32}$$

where $w^{(2)}(x)$ is a polynomial function of x. We then have

$$\oint_C \frac{dz}{2\pi i}\frac{1}{x - z} f(z) = 0\,, \tag{3.33}$$

where now C is a cycle containing all the eigenvalues of Φ_1, but not the eigenvalues m_a and the point x. We then find that $f(x)$ is regular away from $x = m_a$. As it only has at most a single power of ω_2, it will have single poles at $x = m_a$. Near $x = m_a$ it can be approximated

$$f(x) \approx 4S_1S_{2,a}\omega_1(m_a)\frac{1}{x - m_a}\,. \tag{3.34}$$

In fact, this decoupled curve is precisely the one we found in the last section governing the matrix model with a large number of flavors. This shows that indeed as expected the two methods are equivalent.

Let us change some notation appropriate for the present setting. We will denote $S_I = S_{1,I}$, $S^+_{f,a} = S_{2,I}$ and $S^-_{f,a} = S_{12,I}$, and similar notations for the N_I. Let us now see what form the derivatives of the free energy reduce to. First we note that the $a_1 - a_2 = \frac{1}{2}(\tilde{a}_1 - \tilde{a}_2)$, which is half the difference of $\tilde{y}$ on the two sheets. Therefore we can write the variation of the tree level superpotential with respect to the moduli $S_{1,I}$ as

$$\frac{\partial \mathcal{F}_0}{\partial S_I} = \frac{1}{2}\int_{C_I} \tilde{y}(x)dx\,, \tag{3.35}$$

where C_I is the noncompact curve in the decoupled double cover corresponding to $C_{1,I}$, running from Q to P.

For the variation with respect to $S^+_{f,a}$, we note that the $a_2 - a_3 = \frac{1}{2}(\tilde{a}_2 - 3a_3)$. We have $3a_3 = W_1' + 2W_2' + 3S_2\omega_2$, but W_2' is independent of S while the last term vanishes at $S_2 = 0$. Therefore we can write

$$\frac{\partial \mathcal{F}_0}{\partial S^+_{f,a}} = \int_{m_a}^{\infty} (a_2(x) - a_3(x))dx = \frac{1}{2}\int_{m_a}^{Q} (\tilde{y}(x) - W_1'(x))dx + \text{constant}\,. \tag{3.36}$$

We can safely ignore the extra constant for the calculation of the superpotential. In fact it is the contribution of the part of $C_{2,I}$ lying on the decoupled third sheet, and represents the residue of the second node. Similarly, for the degenerate cuts connecting the first and the second sheet we find at $S_2 = 0$

$$\frac{\partial \mathcal{F}_0}{\partial S^-_{f,a}} = \int_{m_a}^{\infty} (a_1(x) - a_3(x))dx = \frac{1}{2}\int_{m_a}^{P} (\tilde{y}(x) - W_1'(x))dx + \text{constant}\,. \tag{3.37}$$

The constants are related to the dynamics of the second node. As we are not interested in these, we can simply substract them. As they do not depend on the glueball expectation values this can be safely done.

For the flavor model we are interested in the limit $S_f = 0$. The only relevant parameters are $S_I \equiv S_{1,I}$. In the expressions above for the derivatives of the free energy we should substitute the solution for $\tilde{y}$ at $S_2 = 0$. This is simply the solution of the 1-matrix model with tree level superpotential $W_1(x)$, and moduli determined by S_I.

The effective superpotential can now be calculated from (3.23) with the proper interpretation of the factors. Note that (3.22) gives the decomposition of the ranks as

$$N_c = \sum_I N_I + \sum_a N^-_{f,a}\,, \qquad N_f = \sum_{a,\pm} N^{\pm}_{f,a}\,. \tag{3.38}$$

Hence we find that the different choices for $\epsilon_a = \pm 1$ give rise to different unbroken flavor symmetries.

3.4 Remarks on Seiberg duality

The A_2 curve has a natural action of the A_2 Weyl group S_3. The part of this group compatible with our limit $\kappa \to \infty$ decoupling the third sheet is a $\mathbb{Z}_2$ symmetry interchanging sheets one and two. Looking at the action of this $\mathbb{Z}_2$ on the cuts they act on the 't Hooft couplings as $\tilde{S}_I = -S_{1,I}$, $\tilde{S}_{2,I} = S_{12,I}$ and $\tilde{S}_{12,I} = S_{2,I}$. The curve after the $\mathbb{Z}_2$ action describes a similar gauge system but with different parameters. In terms of these dual variables the effective superpotential becomes

$$N_I \frac{\partial \mathcal{F}_0}{\partial S_I} + N^+_{f,a}\frac{\partial \mathcal{F}_0}{\partial S^+_{f,a}} + N^-_{f,a}\frac{\partial \mathcal{F}_0}{\partial S^-_{f,a}} = -N_I \frac{\partial \mathcal{F}_0}{\partial \tilde{S}_I'} + N^-_{f,a}\frac{\partial \mathcal{F}_0}{\partial \tilde{S}^+_{f,a}} + N^+_{f,a}\frac{\partial \mathcal{F}_0}{\partial \tilde{S}^-_{f,a}}\,. \tag{3.39}$$

We read off from this that this can be interpreted as the effective superpotential for a dual theory with $\tilde{N}_I = -N_I$ while $\tilde{N}^{\pm}_{f,a} = N^{\mp}_{f,a}$. This implies

$$\tilde{N}_c = \sum_I \tilde{N}_I + \sum_a \tilde{N}^-_{f,a} = -\sum_I N_I + \sum_a N^+_{f,a} = N_f - N_c\,, \quad \tilde{N}_f = N_f\,. \tag{3.40}$$

Note that for this to have a gauge theory interpretation we need $N_f > N_c$, as the number of colors is always positive. This is exactly the transformation of Seiberg's electric-magnetic duality [48]. Indeed this Weyl group symmetry has been connected to Seiberg-like duality in the context of D-branes on Calabi-Yau geometries with ADE geometry in [47].

Note that the quantum curve has two types of degenerate cuts connecting the third "flavor" sheet, corresponding to $\epsilon_a = \pm 1$. These have a different physical interpretation. This can be seen form the behavior of the N_c dependence. The duality described above interchanges these two cuts. In the Seiberg duality, the magnetic model also is slightly different, as it has extra gauge invariant fields, and an extra term in the tree level superpotential. Furthermore the original Seiberg duality has massless quarks, so we need to take a limit $m \to 0$.

4. The gaussian model

4.1 Matrix model calculation

Let us for specifity consider the simplest example of $\mathrm{U}(N_c)$ super Yang-Mills with N_f massive flavors and an adjoint, with a gaussian tree level superpotential for the adjoint. That is, we take

$$W_1(\Phi_1) = \frac{g}{2}\Phi_1^2 \,. \tag{4.1}$$

The superpotential W_2 of the second node will be taken in terms of the masses m_a for the flavors as above.

The tree level superpotential has only a single vacuum at $\Phi_1 = 0$. Hence there will be only a single glueball $S_{1,I} = S$. For $S_2 = 0$ the solution for $y(x)$ is has the well known half circle form,

$$y(x) = \sqrt{W_1'(x)^2 - f(x)} = \sqrt{g^2x^2 - 4gS} \,. \tag{4.2}$$

(We leave out the tilde as we are only considering the decoupled system).

We take the noncompact cycle $C = C_I$, connecting the point running from Q to P passing through the S cut. We will take a regularization where P and Q are taken at $\tilde{\Lambda}$. We then have, at $S_2 = 0$, $S_1 = S$,

$$\frac{\partial \mathcal{F}_0}{\partial S} = \frac{1}{2}\int_C y(x)dx = -S\log\frac{S}{g\tilde{\Lambda}^2} + S \,. \tag{4.3}$$

For the derivative with respect to $S^+_{f,a}$ at $S_f = 0$ we have to take the integral from the point m_a to either P or Q. We find, up to a S independent constant,

$$\begin{aligned}\frac{\partial \mathcal{F}_0}{\partial S^+_{f,a}} &= \frac{1}{2}\int_{m_a}^{Q}(y(x) - W_1'(x))dx \\ &= S\log\left(\frac{m_a}{\tilde{\Lambda}}\right) - \frac{1}{2}S + S\log\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-4\alpha_a S}\right) - \frac{1}{4\alpha_a}\sqrt{1-4\alpha_a S} + \frac{1}{4\alpha_a}\,, \\ &= S\log\left(\frac{m_a}{\tilde{\Lambda}}\right) - \frac{1}{2}S + S\log\gamma_+(\alpha_a S) + \frac{S}{2\gamma_+(\alpha_a S)}\,,\end{aligned} \tag{4.4}$$

where $\alpha_a = 1/gm_a^2$ and we have introduced the functions

$$\gamma_\pm(x) = \frac{1}{2} \pm \frac{1}{2}\sqrt{1-4x}. \tag{4.5}$$

Similarly, we can calculate the derivative with respect to $S_{f,a}^-$, for which we find

$$\begin{aligned} \frac{\partial \mathcal{F}_0}{\partial S_{f,a}^-} &= \frac{1}{2}\int_{m_a}^{P} (y(x) - W_1'(x))dx \\ &= -S\log\left(\frac{S}{gm_a\tilde{\Lambda}}\right) + \frac{1}{2}S + S\log\gamma_-(\alpha_a S) + \frac{S}{2\gamma_-(\alpha_a S)}, \end{aligned} \tag{4.6}$$

From our proposal (2.5), we can give the following expression for the effective superpotential, noting that $N_1 = N_c - N_f^-$ due to (3.22),

$$W_{\text{eff}}(S) = -N_c S\left(\log\frac{S}{\Lambda^3} - 1\right) + \sum_{a=1}^{N_f} S\left(\log\gamma_{\epsilon_a}(\alpha_a S) + \frac{1}{2\gamma_{\epsilon_a}(\alpha_a S)} - \frac{1}{2}\right). \tag{4.7}$$

where $\epsilon_a = \pm 1$ according to whether the integral was taken to Q or P, and the scale Λ is given in terms of the cutoff as $\Lambda^{3N_c} = \tilde{\Lambda}^{2N_c - N_f} g^{N_c} \det m$. This is precisely the form found by an explicit summation of the planar diagrams with up to one boundary [6, 15].

The actual effective superpotential can be found by minimizing the above with respect to S_1, $\partial_S W_{\text{eff}} = 0$. The critical points are determined by the solutions to the equation

$$\left(\frac{S}{\Lambda^3}\right)^{N_c} = \prod_{a=1}^{N_f} \gamma_{\epsilon_a}(\alpha_a S). \tag{4.8}$$

At a critical point the logarithmic terms in the effective superpotential cancel, and it can be written

$$W_{eff,crit} = \frac{2N_c - N_f}{2} S + \sum_{a=1}^{N_f} \frac{S}{2\gamma_{\epsilon_a}(\alpha_a S)}, \tag{4.9}$$

where S is a solution to the equation above.

For simplicity we will now consider the case where all masses are equal to m and all $\epsilon_a = 1$. This gives an equation for S which can be written

$$\left(\frac{S}{\Lambda^3}\right)^{\frac{2N_c - N_f}{N_f}} - \left(\frac{S}{\Lambda^3}\right)^{\frac{N_c - N_f}{N_f}} + \frac{\Lambda^3}{m^2} = 0. \tag{4.10}$$

Here we used that $S = 0$ can not be a solution. The expression for the effective superpotential at the critical point is given by

$$W_{eff,crit} = \frac{\Lambda^3}{2}\left((2N_c - N_f)\frac{S}{\Lambda^3} + N_f\left(\frac{\Lambda^3}{S}\right)^{\frac{N_c - N_f}{N_f}}\right), \tag{4.11}$$

where for S we should insert a solution to the above equation.

4.2 Calculation via meson potential

Another way to calculate the answer is to integrate out Φ and express everything in terms of the meson field $X = \tilde{Q}Q$. This procedure was used in [6] for the case of a single flavor, and in [15] for more flavors. It is useful though for us to review it as it will give an interpretation of the two flavor cuts in terms of meson expectation values.

After integrating out Φ, we obtain a tree level superpotential which is quadratic in terms of the meson field X. To get the full effective superpotential, we need to include the non-perturbative Affleck-Dine-Seiberg potential [49]. We then get an effective superpotential for the meson field of the form

$$W_{\text{eff}} = (N_c - N_f)\left(\frac{\hat{\Lambda}^{3N_c - N_f}}{\det X}\right)^{\frac{1}{N_c - N_f}} + \operatorname{Tr} mX - \frac{1}{2g}\operatorname{Tr} X^2 \,. \tag{4.12}$$

The equation for a critical point of this superpotential is

$$\left(\frac{\hat{\Lambda}^{3N_c - N_f}}{\det X}\right)^{\frac{1}{N_c - N_f}} 1 - mX + \frac{1}{g}X^2 = 0 \,. \tag{4.13}$$

At the critical point we can write the superpotential in a slightly simpler form,

$$W_{eff,crit} = \frac{2N_c - N_f}{2}\left(\frac{\hat{\Lambda}^{3N_c - N_f}}{\det X}\right)^{\frac{1}{N_c - N_f}} + \frac{1}{2}\operatorname{Tr} mX \,. \tag{4.14}$$

More generally, we can diagonalize the meson matrix X. We can actually assume that X is diagonalized simultaneously with the mass matrix m. This follows from the equations of motion for the flavors,

$$0 = \frac{\partial S}{\partial Q} Q - \tilde{Q}\frac{\partial S}{\partial \tilde{Q}} = mX - Xm \,. \tag{4.15}$$

Note that these equations of motions are exact quantum mechanically, as the flavors appear only quadratic in the action. We will therefore call the eigenvalues of the meson field X_a. The equations for the critical points for the eigenvalues can now be written

$$\sigma - m_a X_a + \frac{1}{g}X_a^2 = 0 \,, \tag{4.16}$$

where we have introduced $\sigma = \left(\frac{\hat{\Lambda}^{3N_c - N_f}}{\det X}\right)^{\frac{1}{N_c - N_f}}$. We can now solve for the eigenvalues of the meson field in terms of σ, finding

$$X_a = \frac{\sigma}{m_a \gamma_{\epsilon_a}(\alpha_a \sigma)} \,, \tag{4.17}$$

where $\epsilon_a = \pm$. Due to the obvious relation $\det X = \prod_a X_a$, σ has to satisfy the relation

$$\prod_{a=1}^{N_f} \gamma_{\epsilon_a}(\alpha_a \sigma) = \frac{\sigma^{N_c}}{\hat{\Lambda}^{3N_c - N_f}\det m} \,. \tag{4.18}$$

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We now find that if we relate the scales as $\hat{\Lambda}^{3N_c-N_f} = \frac{\Lambda^{3N_c}}{\det m}$, and we replace σ by S, this is precisely the equation (4.8) for S. Moreover with this identification also the effective superpotentials (4.9) and (4.14) agree.

As the choice of ϵ_a is related to the choice of sheet on which the point corresponding to m_a, we see that the two different branch cuts in the A_2 model near $x = m_a$ correspond to the choice of eigenvalues for the meson field.

5. Conclusions and discussion

We studied effective glueball superpotentials for $\mathcal{N} = 1$ super Yang-Mills theories with fundamental matter and an adjoint Higgs field. We calculated these using matrix models at large N with a large number of flavors. In particular, we used the 2-matrix model in a special limit. As noted earlier, this allows us to deal exactly, and purely in a large-N limit, with arbitrary tree level superpotentials for the adjoint field. Although we only worked out in detail the gaussian case, more general superpotentials are straightforwardly added, but technically more difficult to solve exactly.

The description of flavors using the A_2 quiver matrix model has some intriguing properties. The presence of two types of cuts in the A_2 model connecting the sheet corresponding to the flavor As noted above, we can see a sign of Seiberg-like duality from the $\mathbb{Z}_2$ action interchanging two sheets. It would be interesting to pursue this further.

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